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Design and development of a 3D system for the measurement of tube eccentricity

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Abstract
We present a novel method for the 3D optical measurement of tube eccentricity. The prototype is based on two pairs of laser slits that illuminate the external and internal walls of the tube respectively. Each laser slit captures a 3D semi-profile in the zone close to the cut section of the tube. The laser slits are assembled following a suitably designed layout, which allows us to obtain the circumferential profiles of the internal and external tube surfaces. These profiles are fitted to two circles, and the eccentricity is measured as the distance between their respective centres. The system is suitable for monitoring the wall thickness in correspondence of tube cross sections characterized by scratches and chippings left by the cutting tool in a static way. In this paper, the method and the procedures developed to implement the measurement are described. The characterization of the laser slits, as well as the measurement performance of the system, is detailed. A number of experimental results highlighting the system performance in comparison with a 2D vision approach are discussed.

Keywords: tube eccentricity, 3D vision, camera modelling, 2D vision

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Quality control and material saving are of primary importance in tube production. Eccentricity (i.e. the distance between the centres of the external and internal circular profiles of the tube walls) is a key parameter that needs to be monitored: being a consequence of the drawing process, it should be kept as low as possible for high tube quality. Tube eccentricity is normally measured in two ways: on-line monitoring on a continuous basis during tube production, and off-line, more accurate measurements on an adequate number of tubes sampled from the production.

On-line measurements are commonly performed using ultrasonic techniques [1, 2]. Piezo transducers transmit ultrasonic waves to the tube; the time of flight of the echoes from the two surfaces is measured and is proportional to the wall thickness. These measurements are generally single point: multiple transducers must be generally used to monitor the tube wall geometry (including eccentricity, diameter, ovality and minimum wall); moreover, the transducers/sensors must be acoustically coupled to the tube by means of liquids, and this restricts the measurement to tubes at room temperature, many hours after forming. A non-contact alternative to ultrasonic gauging is represented by laser photoacoustic methods, whereby an ultrashort, intense laser pulse creates an ultrasonic pulse when impinging on the tube surface, and the pulses induce an optical perturbation of the laser beam, detected interferometrically [3]. This technology performs well in the hot production of seamless tubes but is expensive and difficult to operate in a harsh environment.

Off-line measurements of eccentricity on production samples are carried out on tubes along their cross section. A manual approach requires mechanical calipers to measure the maximum and minimum thicknesses, and to calculate the semi-difference between the two values. This approach is rather imprecise, operator dependent and time consuming (measurements must be repeated at different diameters of the tube) [4]. 2D vision systems are an ideal non-contact approach to off-line measurements: an image of the tube end cross section is taken, external and internal profiles of the tube are derived, circularly fitted and the distance between the centres...
is computed [5]. Suitable illumination, camera calibration and telecentric optics are typically used to enhance the visibility of the profiles, and to compensate for lens distortion.

2D vision measurements are highly effective only if the cross sections of the tube are properly finished. To achieve this, scratches and chippings left by the cutting tool must be removed, and this is an additional process that may be incompatible with the requirements for a timely intervention in case of excessive eccentricity. Measurement of the eccentricity near the tube ends without additional wall machining can be performed using laser-based sensors combined with triangulation techniques. Single-point triangulation can be performed on rotating tubes by means of hybrid contact/laser techniques, such as in [6], or by means of laser triangulators, such as in [7], where they measure the profiles along the external and internal circumferences at the ends of the rotating tube. An alternative to the previous system is given in [8], where laser slits at the tube ends, each with a camera acquiring the laser blade at an angle, are used.

From the above discussion, it is evident that high-accuracy, optical, static (i.e. without the need to rotate the tube) tube eccentricity measurements on non-machined tubes would have incredible advantages over the above techniques. A project assigned to our laboratory by a leading brass tube manufacturer focused on the measurement of non-machined tube eccentricity, in order to reject (by subsequent cuttings) the eccentric, initial section of a tube, until non-eccentric sections were found. The required measurement uncertainty was equal to \( \pm 0.02 \) mm. A static approach to the measurement was a further requirement, to avoid expensive modifications of the production line. Achieving this goal would result in a remarkable saving of money for the company, due to the reduction of machining on eccentric tube parts (to be rejected ex-post).

The result of the research performed by our group is a complete, static 3D system described in this paper. It consists of four laser slits, used to project light blades onto the tube cross section. The originally straight shape of each light blade is deformed by the surface shape. This deformation is acquired by four cameras, angled with respect to the projection direction. The 3D shape of the tube points illuminated by the pattern is retrieved by means of optical triangulation [9]. Two laser slits capture the internal right and left semi-profiles: the corresponding laser blades must be angled with respect to the longitudinal axis of the tube, as shown in figures 1(a) and (b).
The remaining two laser slits capture the right and left external semi-profiles, as shown in figures 1(c) and (d).

The eccentricity measurement is obtained by means of suitably designed procedures which (i) measure the semi-profiles, (ii) combine them to form two profiles, corresponding to the external and internal circumferences of the tube walls, (iii) perform the circular fitting of the profiles and evaluate the eccentricity parameter by measuring the distance between their centres.

In this paper, we present the theoretical and experimental activities carried out to develop the 3D system. In section 2, we present the optical layout common to the laser slits and the measurement principle for capturing the semi-profiles. In section 3, we show the geometrical layout of the whole system and the measurement principle developed to obtain the eccentricity parameter. Section 4 is dedicated to show the experimental results.

2. Optical layout of the laser slits and measurement principle

The optical layout common to each laser slit is shown in figure 2. Points oc and op are the entrance and exit pupils of the video camera and the laser source respectively. The camera is represented by an image plane κ. The parameters i, j index the plane columns and rows. The coordinates \(\{x_c, \ y_c, \ z_c\}\) define the camera local reference (CLR) system, with origin at point oc. The axis \(z_c\) intersects the plane κ at the principal point \(o' (i_o, \ j_o)\). The distance \(z_o\) is the focal length \(f_o\).

The laser source is equipped with a cylindrical lens, which expands the light beam along one direction. A plane of light is generated, denoted by \(\lambda\). In figure 2, it intersects the external, left wall of the tube along the semi-profile Lp. The optical axes \(z_p\) and \(z_c\) of the laser and of the camera form an angle \(\chi = 45^\circ\). The coordinates \(\{x, \ y, \ z\}\) define the world reference (WR) system, with origin at point o. In our system, WR is oriented as shown in the figure. Given an object point P, its coordinates within WR are \(\{x_p, \ y_p, \ z_p\}\).

Figure 2. Optical geometry of the laser slits. The external left semi-profile is presented.

Figure 3. Example of signal CM (red points), overlapped with light profile Lp.

The measurement principle is based on optical triangulation. The light profile Lp is imaged at the plane κ: object points belonging to Lp can be viewed as the intersection between their line of sight and plane \(\lambda\). For example, point P impinges on the plane κ at point \(P'\), and the line of sight is PP'. Point P coordinates are estimated provided that (i) values \((i_p, \ j_p)\) of point P' are measured, and (ii) the pose and the orientation of the planes κ and \(\lambda\) are estimated with respect to the WR system. Step (i) is naturally performed by a video camera, by acquiring and elaborating the intensity image of the scene, and step (ii) is carried out by suitably modelling both the camera and the projector [10].

2.1. Image elaboration method

A considerable number of algorithms that perform laser stripe detection are available in the literature: peak detection and skeletonizing techniques have been developed, especially for reducing the influence of laser speckle noise [11–13] as far as possible. In our case, however, a very simple approach to laser slit detection has been implemented, considering that measured points would have been used in a circle fitting algorithm, which would have filtered out residual noise in 3D raw data.

Our image elaboration approach estimates with sub-pixel resolution the value of the coordinate \(i_p\). The following formula is used:

\[
i_p = \frac{\sum_i l_i \cdot i}{\sum_i l_i}.
\] (1)

Equation (1) performs the weighted average of the grey values \(l_i\) of the row \(j_p\). The value \(i_p\) represents the coordinate of the centre of mass (CM) of the laser profile along the row \(j_p\). The coordinate \(j_p\) is the row index of point P'. Equation (1) is applied to all the image rows; the resulting signal is called signal CM. An example of signal CM is shown in figure 3.

2.2. Modelling of the camera

The pose and the orientation of the camera with respect to WR are modelled by using the pin-hole camera model [14]. This model describes the camera with six parameters for its position.
and orientation (extrinsic parameters), and five additional parameters that model the image formation in the presence of a paraxial lens (intrinsic parameters). The extrinsic parameters express the translation vector between points \( o \) and \( o' \), and the rotation matrix between the reference systems WR and CLR. The intrinsic parameters model focal length \( f_x \), coordinates \( j_o, i_o \) of point \( o' \) and scale factors \( s_x, s_y \) along the columns and rows of the image.

The method used to estimate the model parameters is based on the knowledge of a suitable number of points defined in the system WR, whose coordinates are the known terms in the following equations:

\[
\begin{align*}
    i &= s_x \cdot \frac{r_{11}x + r_{12}y + r_{13}z + t_x}{r_{31}x + r_{32}y + r_{33}z + t_z} + i_o \\
    j &= s_y \cdot \frac{r_{21}x + r_{22}y + r_{23}z + t_y}{r_{31}x + r_{32}y + r_{33}z + t_z} + j_o.
\end{align*}
\]

In equations (2), the coefficients \( r_{ij} \), \( i = 1, \ldots, 3 \) define the rotation matrix and \( \{t_x, t_y, t_z\} \) are the components of the translation vector. The parameters \( s_x, s_y, i_o, j_o \) and \( f_z \) are the intrinsic parameters. The coordinates \( \{x, y, z\} \) represent the position of a generic point in the reference system WR. The values \( i \) and \( j \) are the coordinates of the point at the image plane.

Since we have two equations and eleven unknowns, at least six points in the WR must be used. The solution method is well known, and we do not go into the details of the procedure. The interested reader can refer to [15] for further details. Here, it is worth noting that the problem is linear, and a maximum likelihood criterion is used to estimate the unknowns.

In our system, the coordinates of world points are defined by the master object shown in figure 4(a). The master (420 mm in height by 300 mm in width) has the form of a rigidly mounted onto a guide, oriented along the \( Z \) axis. As shown in the figure, the circles’ rows and columns are oriented with respect to the reference system WR. The first position of the master is made to correspond with the lower edge of the measurement range. The guide (PI M-511.5i, with a bidirectional repeatability of 0.5 \( \mu \)m) is PC controlled and moves the master with micrometric precision.

A suitably developed procedure acquires the image of the master, thresholds it and fills the circles, as shown in figure 4(b). Then, the image plane coordinates \( i_x, j_x \) of the centre \( S \) of each circle are calculated. A suitable number of circles is chosen to feed the model. These circles are called ‘calibration markers’. For each calibration marker, both image coordinates \( i_x, j_x \) and world coordinates \( \{x_s, y_s, z_s\} \) are used in equation (2). This procedure is repeated at different positions \( z_o \) of the master along the coordinate \( z \), in order to capture a suitable number of points and to decrease the influence of the inaccuracies of the master and of the image elaboration process. The solution of the over-determined resulting system leads to the determination of the camera parameters.

2.3. Modelling of the projector

The modelling of the projector is aimed at estimating the orientation of the plane \( \lambda \), with respect to the reference system WR. Three parameters must be estimated, i.e. the coefficients \( a, b \) and \( c \) of the following plane equation:

\[
z = ax_s + by_s + c.
\]

In equation (3), \( \{x_s, y_s, z_s\} \) are the coordinates of the points belonging to the plane \( \lambda \). They are expressed in the WR. To estimate the coefficients \( a, b \), and \( c \), the following procedure has been developed. A white sheet of paper is placed on the master in figure 4(a), and for a suitable number of known positions \( z_s \) of the master along \( z \), the laser profile \( L_p \) is acquired. In this case, the profile \( L_p \) is a straight line. It is elaborated by using the procedure presented in section 2.1, which outputs, for each point of signal CM, the coordinates \( i, j \) at the plane \( \kappa \). These coordinates are inserted into equations (2) together with the corresponding values \( z_s \). Since the camera parameters are known, it is possible to solve equations (2) for the unknowns \( x_s \) and \( y_s \). A cloud of points representing the plane \( \lambda \) is obtained. Their coordinates are used to fit the coefficients \( a, b, \)
In equation (3). Since the problem is linear, a simple linear regression algorithm is used [15].

2.4. Execution of the measurement

The measurement of object points is straightforward. Referring to the situation depicted in figure 2, for any object point P, the image plane coordinates \( i_P, j_P \) are measured and used in the system of equations (2) and (3). Since both the camera and the laser plane parameters are known, the system is determined and can be solved for the coordinates \( \{x_P, y_P, z_P\} \).

3. Optical layout of the whole system and the measurement principle of the eccentricity

The optical layout of the whole system is shown in figure 5. The coordinates \( \{X, Y, Z\} \) define the global reference (GR) system, with origin at point O \((X_0, Y_0, Z_0)\). The coordinate \( Y \) is perpendicular to the figure plane. The tube is oriented with its longitudinal axis along \( Z \). The laser slits are denoted by LS1, LS2, LS3 and LS4. LS1 and LS2 capture the right and the left external semi-profiles respectively. LS3 and LS4 gauge the left and the right internal semi-profiles respectively. The laser blades are denoted by LB1, LB2, LB3 and LB4. They are perpendicular to the figure plane and intersect each other on the \( Y \) axis. The angles \( \gamma_k \) \((k = 1, 2, 3, 4)\) define the positions of the laser blades with respect to \( X \). The angles \( \gamma_3 \) and \( \gamma_4 \) are set in such a way that LB3 and LB4 impinge on a portion of the tube where the eccentricity can be considered constant along the \( Z \) direction. The angles \( \gamma_1 \) and \( \gamma_2 \) are set so that laser blades LB2 and LB1 belong to the same plane as LB3 and LB4 respectively. The angles \( \alpha_k \) define the position of the optical axis of each camera in laser slit LS_k with respect to the direction \( X \). From the figure, we obtain \( \alpha_1 = \chi - \gamma_1 \), \( \alpha_2 = \chi - \gamma_2 \), \( \alpha_3 = \chi + \gamma_3 \) and \( \alpha_4 = \chi + \gamma_4 \).

Each semi-profile \( L_P \) is measured within the reference system WR_k of the laser slit LS_k, defined by the modelling procedures shown in sections 2.2 and 2.3. For example, the external left semi-profile is measured within the reference system WR_2 \( \{x_2, y_2, z_2\} \) of the laser slit LS_2.

Point \( o_2 \) is the origin of WR_2 and corresponds to the entrance pupil of the video camera; its coordinates are computed from knowledge of the coordinates \( \{t_x, t_y, t_z\} \) in equations (2).

The measurement principle developed to obtain the eccentricity is shown in figure 6. The first step is to map the semi-profiles \( L_{P1} \) and \( L_{P2} \) in the reference systems WR_2 and WR_3 and to combine them with profiles \( L_{P2} \) and \( L_{P3} \) respectively. The output profiles are denoted by \( L_{Pext} \) and \( L_{Pint} \). \( L_{Pext} \) is formed by the external right and left semi-profiles both expressed in the reference system WR_2. \( L_{Pint} \) is formed by the internal right and left semi-profiles both expressed in the reference system WR_3.

The second step is to map \( L_{Pext} \) and \( L_{Pint} \) onto the planes \( XY_2 \) and \( XY_3 \), respectively, to compensate for the angled illumination of the walls, which is necessary to inspect the internal surface of the tube. The resulting profiles are denoted by \( L_{PE} \) and \( L_{Ptot} \). The third step is to map the profile \( L_{Ptot} \) onto the plane \( XY_2 \), in order to align it with the profile \( L_{PE} \). The output profile is \( L_{P} \). In step 4, both \( L_{PE} \) and \( L_{P} \) are fitted to a circle. Eccentricity \( Em \) is evaluated as the distance between the centres \( C_E \) and \( C_I \) of the two circumferences. Each step is detailed in the following sections.

3.1. The procedure developed in step 1

The aim of developing this procedure is to express the semi-profiles \( L_{P1} \) and \( L_{P4} \) in reference systems WR_2 and WR_3 respectively. The mapping is described by the following relationships:

\[
\begin{bmatrix}
x_{2P} \\
y_{2P} \\
z_{2P}
\end{bmatrix} = R_{21} \cdot \begin{bmatrix}
x_{1P} \\
y_{1P} \\
z_{1P}
\end{bmatrix} + t_{12}
\]

\[
\begin{bmatrix}
x_{1P} \\
y_{1P} \\
z_{1P}
\end{bmatrix} = \begin{bmatrix}
\cos(\alpha_1 + \alpha_2) & 0 & -\sin(\alpha_1 + \alpha_2) \\
0 & 1 & 0 \\
\sin(\alpha_1 + \alpha_2) & 0 & \cos(\alpha_1 + \alpha_2)
\end{bmatrix} \cdot \begin{bmatrix}
x_{1I} \\
y_{1I} \\
z_{1I}
\end{bmatrix} + \begin{bmatrix}
x_{1I2} \\
y_{1I2} \\
z_{1I2}
\end{bmatrix}
\]

(4)
for $L_{p1}$, and
\[
\begin{bmatrix}
{x_{3f}} \\
{y_{3f}} \\
{z_{3f}}
\end{bmatrix}
= R_{4,3} \cdot \begin{bmatrix}
{x_{4f}} \\
{y_{4f}} \\
{z_{4f}}
\end{bmatrix} + t_{4,3}
\]
\[
= \begin{bmatrix}
-\cos(\alpha_3 + \alpha_4) & 0 & \sin(\alpha_3 + \alpha_4) \\
0 & -1 & 0 \\
-\sin(\alpha_3 + \alpha_4) & 0 & -\cos(\alpha_3 + \alpha_4)
\end{bmatrix} \cdot \begin{bmatrix}
{x_{4f}} \\
{y_{4f}} \\
{z_{4f}}
\end{bmatrix} + \begin{bmatrix}
x_{4,3} \\
y_{4,3} \\
z_{4,3}
\end{bmatrix}
\] (5)

for $L_{p4}$.

The coordinates $\{x_{1P}, y_{1P}, z_{1P}\}$ in equation (4) are the 3D coordinates of a generic point $P$ of the semi-profile $L_{p1}$. The coordinates $\{x_{2P}, y_{2P}, z_{2P}\}$ express the position of $P$ in the reference system $W_{R2}$. $R_{1,2}$ and $t_{1,2}$ are the rotation matrix and the translation vector between the reference systems $W_{R1}$ and $W_{R2}$.

The coordinates $\{x_{4f}, y_{4f}, z_{4f}\}$ in equation (5) are the coordinates of a generic point $T$ of the semi-profile $L_{p4}$, while $\{x_{3f}, y_{3f}, z_{3f}\}$ express the position of $T$ in the reference system $W_{R1}$. $R_{4,3}$ and $t_{4,3}$ are the rotation matrix and the translation vector between the reference systems $W_{R4}$ and $W_{R3}$.

The rotation matrices $R_{1,2}$ and $R_{4,3}$ are known, since the angles $\alpha_k$ are measured during the system setup. In contrast, the translation vector components $\{x_{1,2}, y_{1,2}, z_{1,2}\}$ and $\{x_{4,3}, y_{4,3}, z_{4,3}\}$ must be estimated.

To this end, the master object shown in figure 7 is used. $N$ circles are drawn on a transparent film and positioned in the central part of the master. Each circle has a diameter equal to 3 mm, and the distance between adjacent circles is equal to 6 mm.

3.1.1. Estimation of the components $\{x_{1,2}, y_{1,2}, z_{1,2}\}$. To estimate the components $\{x_{1,2}, y_{1,2}, z_{1,2}\}$, the master is oriented with the coordinates $x_M$ and $y_M$ parallel to $Z$ and $Y$, respectively, and point $O_M$ at point $O$. In this way, both laser slits $LS_1$ and $LS_2$ simultaneously view the circles and can measure the position of the circle centres (centroids) within their respective reference systems.

If we consider a single measured centroid, for example point $R$ in figure 8(a), the coordinates $\{x_{1R}, y_{1R}, z_{1R}\}$ of the reference system $W_{R1}$ represent its position with respect to the reference systems $W_{R4}$ and $W_{R3}$, respectively. The translation vector $t_{1,2}$ equals segment $\overrightarrow{OM}$.

Denote by $\{x_{1,2}^R, y_{1,2}^R, z_{1,2}^R\}$ the estimates of the translation vector obtained from the coordinates of point $R$; they can be calculated by the geometrical construction presented in figure 8(a) as follows:
\[
\begin{align*}
x_{1,2}^R &= o_1 D = D_{CA} + o_1 A \\
&= x_{2R} + z_{1R} \cdot \sin(\alpha_1 + \alpha_2) + x_{1R} \cdot \cos(\alpha_1 + \alpha_2) \\
y_{1,2}^R &= y_{1R} \\
z_{1,2}^R &= o_2 D = F_{RE} + o_2 A \\
&= z_{2R} + z_{1R} \cdot \cos(\alpha_1 + \alpha_2) - x_{1R} \cdot \sin(\alpha_1 + \alpha_2),
\end{align*}
\] (6)

Equations (6) hold for all the centroids acquired by the laser slits $LS_1$ and $LS_2$. The values $\{x_{1,2}, y_{1,2}, z_{1,2}\}$ of the translation vector $t_{1,2}$ are obtained by averaging the components evaluated for the $N$ centroids of the master.
3.1.2. Estimation of the components \{tx_{4,3}, ty_{4,3}, tz_{4,3}\}. To estimate the components \{tx_{4,3}, ty_{4,3}, tz_{4,3}\}, the master shown in figure 7 is rotated by 90° so that the coordinate \textit{x}\textit{y}\textit{z}\textit{d} is parallel to \textit{X}, and the circles are viewed by the laser slits \textit{LS3} and \textit{LS4} simultaneously. In this case, the geometry used to perform the calculations is shown in figure 8(b), where point \textit{Q} plays the same role as point \textit{R} in figure 8(a), and the translation vector \textit{t}_{4,3} equals segment \textit{OQ}.

Denote by \{tx_{4,3}^Q, ty_{4,3}^Q, tz_{4,3}^Q\} the estimates of the translation vector obtained from the coordinates \{x_{4Q}, y_{4Q}, z_{4Q}\} and \{x_{3Q}, y_{3Q}, z_{3Q}\} of point \textit{Q}; they are calculated by means of the following relationships:

\[
\begin{align*}
tx_{4,3}^Q &= \text{OX} = \text{AX} - \text{BE} + \text{EF} \\
&= x_{4Q} - z_{4Q} \cdot \sin(\alpha_3 + \alpha_4) + x_{4Q} \cdot \cos(\alpha_3 + \alpha_4) \\
ty_{4,3}^Q &= y_{4Q} - y_{4Q} \\
tz_{4,3}^Q &= \text{OY} = \text{QD} + \text{BQ} + \text{AB} \\
&= z_{4Q} + z_{4Q} \cdot \cos(\alpha_3 + \alpha_4) + x_{4Q} \cdot \sin(\alpha_3 + \alpha_4).
\end{align*}
\]

Equations (7) are applied to all the master centroids, and the vector \textit{t}_{4,3} components are estimated by averaging the resulting values.

3.1.3. Evaluation of the profiles \textit{Lp}_{ext} and \textit{Lp}_{int}. The profiles \textit{Lp}_{ext} and \textit{Lp}_{int} are formed by the semi-profiles obtained by using the values \{tx_{1,2}, ty_{1,2}, tz_{1,2}\} and \{tx_{4,3}, ty_{4,3}, tz_{4,3}\} in equations (4) and (5), respectively, and by considering the semi-profiles \textit{Lp}_2 and \textit{Lp}_3 in \textit{Lp}_{ext} and \textit{Lp}_{int}, respectively.

3.2. The procedure developed in step 2

This procedure is aimed at projecting the profile \textit{Lp}_{ext} onto the plane \textit{XY}_2, and the profile \textit{Lp}_{int} onto the plane \textit{XY}_3.

3.2.1. Projection of \textit{Lp}_{ext} onto the plane \textit{XY}_2. Figure 9(a) shows the geometry used to map the profile \textit{Lp}_{ext} onto the plane \textit{XY}_2, with reference to points \textit{P} and \textit{T}. The aim of developing this procedure is to derive the length of segments \textit{OE} and \textit{OA} for points \textit{P} and \textit{T}, respectively, using their coordinates in the reference system \textit{WR}_2. From the figure, segment \textit{OE} can be obtained as

\[
\text{OE} = \text{CE} - \text{CO},
\]

where

\[
\text{CE} = \text{DP} \cdot \cos \gamma_2 = \frac{z_P}{\cos(\alpha_2 + \gamma_2)} \cdot \cos \gamma_2.
\]

Segment \textit{CO} is obtained as

\[
\text{CO} = \text{DO} \cdot \cos \gamma_2 = \frac{\overline{OQ}}{\cos(\alpha_2 + \gamma_2)} \cdot \cos \gamma_2.
\]

Segment \textit{OA} is obtained as

\[
\text{OA} = \text{BA} - \text{BO},
\]

where

\[
\text{BA} = \text{IL} = \text{IT} \cdot \cos \gamma_1 = \frac{z_T}{\cos(\gamma_1 - \alpha_2)} \cdot \cos \gamma_1.
\]

The angles \gamma_2, \gamma_1 and \alpha_2 are known. The value of segment \overline{OQ} is equal to the coordinate \textit{z}_{2O} of point \textit{O} along \textit{z}_2. It can be estimated by observing that point \textit{O} represents the intersection between lines \textit{PO} and \textit{TO} (dotted lines in figure 9(a)). The procedure is based on the following steps. First, the equations of lines \textit{PO} and \textit{TO} are obtained from the projections of the left and right components of \textit{Lp}_{ext} onto the plane \textit{x}_2z_2 by means of a linear fitting. Then, the linear system formed by the equations of the two lines is solved with respect to the unknowns \textit{x}_{2O} and \textit{z}_{2O}.

The relationships above are obviously applied to all the points of the profile \textit{Lp}_{ext}; the resulting profile is \textit{Lp}$_E$. 

\[
\text{BO} = \text{KO} \cdot \cos \gamma_1 = \frac{\overline{OQ}}{\cos(\gamma_1 - \alpha_2)} \cdot \cos \gamma_1.
\]

Figure 9. Implementation of step 2. (a) Geometry used to map the profile \textit{Lp}_2 onto the plane \textit{XY}_2. (b) Geometry used to map the profile \textit{Lp}_4 onto the plane \textit{XY}_3.
Figure 10. Image of the system prototype.

Figure 11. Characterization of the laser slits. (a) Plots of the input–output characteristic curves of the laser slits. (b) Plots of differences $z_{nom} - E_k$ versus $z_{nom}$ of the laser slits.

3.2.2. Projection of $L_{p_{int}}$ onto the plane $Xy_3$. Figure 9(b) shows the geometry used to map the profile $L_{p_{int}}$ on the plane $Xy_3$, with reference to points $P'$ and $T'$. The aim of developing this procedure is to obtain the length of segments $OE'$ and $OA'$ for points $P'$ and $T'$, respectively, using their coordinates in

Figure 12. Tube sample 'A': (a) finished side and (b) non-finished side.
the reference system \( \{x_3, y_3, z_3\} \). By looking at the geometry shown in the figure, it is easy to evaluate segment \( \overline{OE} \) as

\[
\overline{OE} = \frac{BP}{\sin \chi} \cdot \cos \gamma_3 = \frac{x_{P'}}{\sin \chi} \cdot \cos \gamma_3. \quad (14)
\]

Segment \( \overline{OA} \) is obtained as

\[
\overline{OA} = \frac{CT}{\sin (\gamma_4 + \alpha_3)} \cdot \cos \gamma_4 = \frac{x_T}{\sin (\gamma_4 + \alpha_3)} \cdot \cos \gamma_4. \quad (15)
\]

Equations (14) and (15) are applied to all the points of the profile \( L_{p_{\text{int}}} \).

### 3.3. The procedure developed in step 3

The aim of developing this step is to map \( L_{p_{\text{int}}} \) onto the plane \( Xy_2 \), so that the resulting profile \( L_{p_I} \) can be expressed in the same reference as \( L_{p_E} \). The developed procedure is very simple, since the plane \( Xy_3 \) is parallel to the plane \( Xy_2 \), and only the translation term of point \( o_3 \) to point \( o_2 \) along \( y_2 \) must be evaluated. To do this, the master shown in figure 7 is positioned with \( O_M = O \), at an angle \( \gamma_3 = \gamma_2 = 45^\circ \) with respect to the \( X \) axis, in such a way that both laser slits \( LS_2 \) and \( LS_3 \) can measure the position of the centroid of each marker. Then, for each \( l \)th centroid, the coordinates \( y_{2l} \) and \( y_{3l} \) are used in the following formula:
The translation component $\Delta y$ is then added to the coordinate along $y_3$ of the points of profile $Lp'_m$; the resulting profile is $Lp_I$.

### 3.4. The procedure developed in step 4

In this step, the profiles $Lp_E$ and $Lp_I$ are fitted to two circumferences. Denoting by $C_0(X_{CO}, y_{2CO})$ and $C_1(X_{CI}, y_{2CI})$ the centres of the circumferences, the measured eccentricity $Em$ is evaluated as

$$Em = \sqrt{(X_{CO}^2 - X_{CI}^2) + (y_{2CO}^2 - y_{2CI}^2)}.$$  \hfill (17)

### 4. Experimental results

The prototype developed to test the technique is shown in figure 10. Four laser slits are assembled, each with a video camera, model IDS UI-1540SE (resolution 1280 $\times$ 1024 pixels), and a laser blade projector (Lasiris Mini 660, 10 mW). The tube is positioned on a suitably developed structure, which orients the longitudinal axis of the tube along the Z axis. The laser blades are positioned according to the layout in figure 5. This operation is not critical, since their mutual intersection, which defines the $Y$ axis, is well visible. The angles $\gamma_k$ are measured by means of a Bosch DWM 40 L angle measurer, with 0.1° of precision. Typical values of angles $\gamma_k$ are 30°. Each video camera is positioned 320 mm from the $Y$ axis. The diameter of the tubes that can be measured by this setup spans from 30 to 80 mm.

#### 4.1. Characterization of the laser slits

The first set of experimental results deals with the performance of the laser slits. For each laser slit $LS_k$, the master shown in figure 4 has been moved along the direction $z_k$ for a measurement range equal to 45 mm, at steps of 1 mm. The lower edge of the range was selected in correspondence with the $Y$ axis. At each position $z_{nom}$ of the master, the laser profile $Lp_k$ has been measured. Then, the coordinates $\{z_k\}$ of the points of $Lp_k$ have been averaged and the resulting mean value $z_k$ has been taken as the estimate $E_k[z_{nom}]$ of the distance $z_{nom}$. The results are summarized in figure 11. Figure 11(a) plots $E_k[z_{nom}]$ versus $z_{nom}$ ($k = 1, 2, 3, 4$): it shows that the four plots overlap, and that they are linear, with angular coefficient equal to 1. Figure 11(b) shows differences $z_{nom} - E_k[z_{nom}]$ versus $z_{nom}$. The values span from $-0.010$ to $0.008$ mm for all the laser slits. Standard deviations $\sigma_{LS_k}$ are $\sigma_{LS1} = \sigma_{LS2} = 0.003$ mm and $\sigma_{LS3} = \sigma_{LS4} = 0.004$ mm.

#### 4.2. Characterization of the whole system

To characterize the whole system, a considerable number of experiments has been performed. Among them, the results related to three tube samples are shown. Their geometric parameters are summarized in table 1; for each one, the nominal external diameter and nominal wall thickness are given. All the samples have been finished at one cross section, to eliminate the effects of the cutting tool. As an example, figure 12(a) shows the machined tube end of sample ‘A’;
its regularity is evident, especially when it is compared to
the other end of the tube, shown in figure 12(b), which has
been left unchanged after the cut. The clear cross section
of the samples has been used to provide values $Ed$ in table 1.
$Ed$ is estimated as the average value of 30 measurements of
eccentricity performed by means of a contact caliper with a
precision of 0.01 mm. We used values $Ed$ as the reference
for the measurement of eccentricity $Em$ in correspondence
with the non-machined side of the samples. Special care was
taken in the production of the samples, to guarantee samples
at constant eccentricity.

For each sample, the measurement procedure described
in section 3 was carried out. Referring to sample ‘A’,
figure 13(a) shows the profiles $Lp_{ext}$ and $Lp_{int}$, figure 13(b)
shows the profiles $Lp_E$ and $Lp_{int}$, and figure 13(c) visualizes
the profiles $Lp_E$ and $Lp_I$, the fitted circles and their respective
centres.

The measured eccentricity was evaluated for ten positions
of the samples on the tube structure. Each position was
obtained by manually rotating the tube. The results are
shown in table 2, which presents the parameter $Em$ at each
measurement and for the three samples. The mean values over
the distribution of the measured $Em$ are equal to the values $Ed$
for each sample, and the measurement uncertainty, expressed
by the standard deviations in the last row of the table, is within
the input requirement ($\pm 0.02$ mm).

We have compared the data in table 2 with those achieved
by means of a 2D vision system, purposely developed by
us to appreciate the quality of the measurement achieved
by means of the 3D system [16]. The system is composed
of a video camera, model IDS uEye 1540-M, with 16 mm
optics, positioned 530 mm far from the tube end. Proper
illumination was used to increase the visibility of the walls
in correspondence with the cutting sections. Suitable 2D
elaboration was carried out on the acquired images: it was
based on image enhancement, edge detection and circular
fitting of the external and internal profiles visible in the images.
The eccentricity was evaluated as the distance between the
centres of the two circles. An example of the elaborated
images is shown in figure 14. They well evidence the surface
scratches, as well as the irregularities of the profiles, which
lead to variations of the wall thickness due to the cutting tool.

We measured the eccentricity $Em$ for each sample,
corresponding to ten different positions of the tubes, obtained
by manually rotating them. The results are summarized in
table 3. The mean values over the distribution of the measured
$Em$ show a significant difference with respect to the values

### Table 1. Parameters of the tube samples used to characterize the system.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Nominal external diameter (mm)</th>
<th>Nominal wall thickness (mm)</th>
<th>$Ed$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>39.00</td>
<td>3.00</td>
<td>0.34</td>
</tr>
<tr>
<td>B</td>
<td>52.00</td>
<td>1.75</td>
<td>0.02</td>
</tr>
<tr>
<td>C</td>
<td>56.00</td>
<td>2.50</td>
<td>0.26</td>
</tr>
</tbody>
</table>

### Table 2. Eccentricity $Em$ measured by the 3D system. Measurements are in mm.

<table>
<thead>
<tr>
<th>Measurement number</th>
<th>Sample ‘A’</th>
<th>Sample ‘B’</th>
<th>Sample ‘C’</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.31</td>
<td>0.02</td>
<td>0.27</td>
</tr>
<tr>
<td>2</td>
<td>0.31</td>
<td>0.02</td>
<td>0.28</td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
<td>0.02</td>
<td>0.24</td>
</tr>
<tr>
<td>4</td>
<td>0.34</td>
<td>0.03</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>0.37</td>
<td>0.03</td>
<td>0.27</td>
</tr>
<tr>
<td>6</td>
<td>0.37</td>
<td>0.03</td>
<td>0.28</td>
</tr>
<tr>
<td>7</td>
<td>0.35</td>
<td>0.03</td>
<td>0.25</td>
</tr>
<tr>
<td>8</td>
<td>0.31</td>
<td>0.02</td>
<td>0.25</td>
</tr>
<tr>
<td>9</td>
<td>0.32</td>
<td>0.02</td>
<td>0.26</td>
</tr>
<tr>
<td>10</td>
<td>0.36</td>
<td>0.01</td>
<td>0.21</td>
</tr>
</tbody>
</table>

| Mean               | 0.34       | 0.02       | 0.26       |
| Std dev.           | 0.02       | 0.01       | 0.02       |

### Table 3. Eccentricity $Em$ measured by the 2D system. Measurements are in mm.

<table>
<thead>
<tr>
<th>Measurement number</th>
<th>Sample ‘A’</th>
<th>Sample ‘B’</th>
<th>Sample ‘C’</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.33</td>
<td>0.25</td>
<td>0.59</td>
</tr>
<tr>
<td>2</td>
<td>0.33</td>
<td>0.19</td>
<td>0.61</td>
</tr>
<tr>
<td>3</td>
<td>0.33</td>
<td>0.19</td>
<td>0.70</td>
</tr>
<tr>
<td>4</td>
<td>0.40</td>
<td>0.20</td>
<td>0.64</td>
</tr>
<tr>
<td>5</td>
<td>0.31</td>
<td>0.19</td>
<td>0.52</td>
</tr>
<tr>
<td>6</td>
<td>0.32</td>
<td>0.26</td>
<td>0.55</td>
</tr>
<tr>
<td>7</td>
<td>0.35</td>
<td>0.17</td>
<td>0.68</td>
</tr>
<tr>
<td>8</td>
<td>0.19</td>
<td>0.19</td>
<td>0.60</td>
</tr>
<tr>
<td>9</td>
<td>0.26</td>
<td>0.19</td>
<td>0.59</td>
</tr>
<tr>
<td>10</td>
<td>0.31</td>
<td>0.23</td>
<td>0.60</td>
</tr>
</tbody>
</table>

| Mean               | 0.31       | 0.20       | 0.61       |
| Std dev.           | 0.05       | 0.03       | 0.05       |
Ed in each sample, in particular for samples ‘B’ and ‘C’, where the difference equals 0.19 and 0.35 mm, respectively. This behaviour is related to the systematic error introduced by the deformation of the circumferential borders, due to the cutting tool. The values of standard deviation are also higher in table 3 than those in table 2. This is by no means surprising, considering the irregularities of the surface at the end of the tubes.

These results are summarized in the plots shown in figure 15 that represent the difference values Em – Ed provided by the 3D system and the 2D system, for each sample.

5. Conclusions

In this paper, the main features of a 3D system for the measurement of tube eccentricity have been presented. The system is designed to perform the measurement at the cutting cross section of the tube end, avoiding rotation of the tube. The experimental results highlight the advantage of using the proposed method, which allows us to compensate for the systematic error due to irregular cuts of the tube samples. The proposed approach is suitable for applications where low-cost, early detection of out of tolerance values of tube eccentricity is required.

Further optimization of the system will deal with the development of a compact layout, based on the use of two cameras and two lasers, and with the extension of the range of the tube diameters which can be measured by the system.

References